PARCIALES ECUACIONES DIFERENCIALES

(1) $y''' + 4y' = 3\,\text{Sen}(2x) + x\cos(2x)$

Una Yp sin hallar constantes

$M^3 + 4M = 0$

$M(M^2+4) = 0$

$M^2 = -4$

$M = \pm 2i$

$y_c(x) = C_1 + C_2\cos(2x) + C_3\,\text{Sen}(2x)$

$$\boxed{Y_p(x) = X(AX+B)\cos(2x) + X(CX+D)\,\text{Sen}(2x)}$$

(2)

$y(x) = e^{-2x}\left(C_1\cos(\ln(3x)) + C_2\,\text{sen}(\ln(3x))\right)$

$M = -2 \pm 3i$

Ejercicio mal diseñado

Propuesta real

$Y(x) = X^{-2}\left(C_1\cos(3\ln(x)) + C_2\,\text{Sen}(3\ln(x))\right)$

$(M+2-3i)(M+2+3i)$

$(M+2)^2 - 9i^2 \qquad (M+2)^2 + 9$

$m^2 + 2m + 4 + 9$

$m^2 + 2m + 13$

$m^2 - m + 2m + m + 13$

$m(m-1) + 3m + 13$

$a = 1 \qquad b = 3 \qquad = c = 13$

$$\boxed{x^2 y'' + 3xy' + 13y = 0}$$

③ El valor de β

$9y'' - y = 0 \qquad y(0) = 2$

$y'(0) = \beta$

$t \to \infty$?

$9m^2 - 1 = 0$

$m^2 = \frac{1}{9}$

$m = \pm \frac{1}{3}$

$Y(x) = C_1 \, e^{x/3} + C_2 \, e^{-x/3}$

$$2 = C_1 + C_2$$

$$\beta = C_1 \frac{1}{3} e^{x/3} - \frac{1}{3} C_2 e^{-x/3} \qquad C_1 = 0$$

$$\beta = \frac{1}{3} C_1 - \frac{1}{3} C_2 \qquad C_2 = 2$$

$$\cancel{\beta = \frac{1}{3} (C_1 - C_2)} \qquad \frac{1}{3}(-2)$$

$$\cancel{\beta = \frac{1}{3} (C_1 - 2C_1)} \qquad \boxed{\boxed{\frac{-2}{3}} = \beta}$$

$$\cancel{\beta = \frac{1}{3} (-C_1)}$$

4

$$W = t^2 e^t$$

$$W(y_1, y_2) = c e^{-\int P(x) dx}$$

$$f(t) = t$$

$$x^2 e^x = c e^{-\int P(x) dx}$$

$$\begin{vmatrix} t & g(x) \\ 1 & g'(x) \end{vmatrix}$$

$$e^{-\int \frac{1}{t} dt}$$

$$e^{-\ln(t)}$$

$$\frac{1}{t}$$

$$t g'(x) - g(x) = t^2 e^t$$

$$\frac{g}{t} = \int \frac{t e^t}{t}$$

$$g' - \frac{g}{t} = t e^t$$

$$\frac{g}{t} = e^t + c \qquad k = ct$$

$$g = e^t t + K$$

(5)

$\frac{3}{2}U'' + KU = 0$

$U(0) = 2$ $\qquad$ $K = ?$

$U'(0) = V$ $\qquad$ $V = ?$

$T = \pi$

$A = 3$

$\omega = 2$

$\frac{2\pi}{\omega} =$

$\cancel{\sqrt{\frac{2K}{3}}} = 2$

$U'' + \frac{2}{3}U = 0$

$\cancel{2K = 6^3}$

$m^2 + \frac{2}{3}K = 0$

$\sqrt{\frac{2K}{3}} = 2^2$

$m = \pm\sqrt{\frac{2K}{3}}\, i$

$2K = 12$

$K = 6$

$2 = C_1 \cos\left(\sqrt{\frac{2K}{3}}t\right) + \cancel{C_2 \operatorname{Sen}\left(\frac{\sqrt{2K}}{3}t\right)} \to 0$

$2 = C_1 \cancel{\cos(2t)} \to 1 + \cancel{C_2 \operatorname{Sen}(2t)} \to 0$

$2 = C_1$

$T = \frac{2\pi}{\mu}$ $C_1 = 2$

$3 = \sqrt{2^2 + c^2}$

$9 = 4 + c_2^2$

$5 = c^2$

$c_2 = \sqrt{5}$

$-\cancel{\operatorname{sen}(2t)}^{\,0}\, c_1(2) + 2c_2 \cancel{\cos(2t)}^{\,1}$

$V = 2c_2$

$\boxed{V = 2\sqrt{5}}$

⑥ $(x^2 - 6x + 9)y'' - (x-3)y' + y = x - 3 \qquad x > 3$

$(x-3)^2 y'' - (x-3)y' + y = x - 3$

$m(m-1) - m + 1 = 0 \qquad y = (x-3)^m$

$m^2 - m - m + 1 = 0 \qquad y' = m(x-3)^{m-1}$

$m^2 - 2m + 1 = 0 \qquad y'' = m(m-1)(x-3)^{(m-2)}$

$(m-1)^2$

$m_1 = 1$

$m_2 = 1$

$\cancel{y_c(x) = C_1 x + C_2 \ln(}$

$$y_c(x) = C_1 (x-3) + C_2 (x-3) \ln(x-3)$$

$$g(x) = \frac{1}{x-3}$$

$$y_p(x) = u_1 (x-3) + u_2 (x-3) \ln(x-3)$$

$$W = \begin{vmatrix} x-3 & (x-3)\ln(x-3) \\ 1 & \ln(x-3) + \dfrac{\cancel{(x-3)}}{\cancel{(x-3)}} \end{vmatrix}$$

$\cancel{\ln((x-3))[(x-3)] - \ln(x-3)}$

$$\frac{\cancel{\ln(x-3)[x-3]} + [x-3] - \cancel{[x-3]\ln(x-3)}}{x-3}$$

$$u_1' = \frac{\begin{vmatrix} 0 & (x-3)\ln(x-3) \\ \dfrac{1}{x-3} & \ln(x-3)+1 \end{vmatrix}}{W}$$

$$\int \frac{\ln(x-3)}{x-3}\,dx$$

$$u_2' = \frac{\begin{vmatrix} x-3 & 0 \\ 1 & \frac{1}{x-3} \end{vmatrix}}{W}$$

$$u = \ln(x-3)$$

$$du = \frac{1}{x-3}\,dx$$

$$u_2' = \frac{1}{x-3}$$

$$\int u\,du$$

$$\int \frac{1}{x-3}\,dx$$

$$\frac{u^2}{2} \qquad \frac{\ln^2(x-3)}{2} = u_1$$

$$u_2 \ln(x-3)$$

$$\frac{1}{2} + \frac{12}{2}$$

$$y_p(x) = \frac{\ln^2(x-3)\,[x-3]}{2} + \ln^2(x-3)\,[x-3]$$

$$y_p(x) = \cancel{2\{\ln^2(x-3)[x-3]\}}$$

$$c_1(x-3) + c_2(x-3) + \frac{3[x-3]}{2}\,[\ln^2(x-3)]$$

$$y(x) = [x-3]\left[c_1 + c_2 + \frac{3[\ln^2(x-3)]}{2}\right]$$

7 Halle la otra solucion Utilizando U de parametro

$$x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0 \qquad x > 0$$

$$y_1 = x^{-1/2} \operatorname{sen}(x)$$

$$y_2 = x^{-1/2} \cdot \operatorname{sen}(x) \int \frac{e^{-\int p(x)dx}}{x^{-1} \operatorname{sen}^2(x)}$$

$$x^{-1/2} \operatorname{sen}(x) \int \frac{1}{\operatorname{sen}^2(x)} dx$$

$$\int \csc^2(x)\, dx$$

$$-\cot x$$

$$-x^{-1/2} \cancel{\operatorname{sen}(x)} \cdot \frac{\cos(x)}{\cancel{\operatorname{sen}(x)}}$$

$$\boxed{-x^{-1/2} \cos(x)}$$

Resorte

(8)

$$\frac{64}{32} = 2 \qquad 64 = K8$$

$$K = 8$$

$m=2$
$K=8$
$\beta = \beta$

$x(0) = -1$
$x'(0) = 5$

$$\text{PVI} \begin{cases} 2x'' + \beta x' + 8x = 0 \\ x(0) = -1 \\ x'(0) = 5 \end{cases}$$

Criticamente amortiguado?

$$\frac{-\beta \pm \sqrt{\beta^2 - 4(2)(8)}}{4}$$

$$\frac{-\beta}{4} \pm \frac{\sqrt{\beta^2 - 64}}{4}$$

$$\beta^2 - 64 = 0$$

$$\beta^2 = 64$$

$$\boxed{\beta = 8}$$

$$2r^2 + r8 + 8 = 0$$

$$r^2 + 4r + 4 = 0$$

$$\frac{-4 \pm \sqrt{16-16}}{2}$$

$$r = -2$$

$$-1 = C_1$$

$$x'(t) = -2C_1 e^{-2t} + C_2 e^{-2t} \cancel{-2C_2 t e^{-2t}}$$

$$5 = 2 + C_2$$

$$C_2 = 3$$

$X(t) = \cancel{e}$

$$X(t) = C_1 e^{-2t} + t e^{-2t} C_2$$

$$x(t) = -e^{-2t} + 3te^{-2t}$$

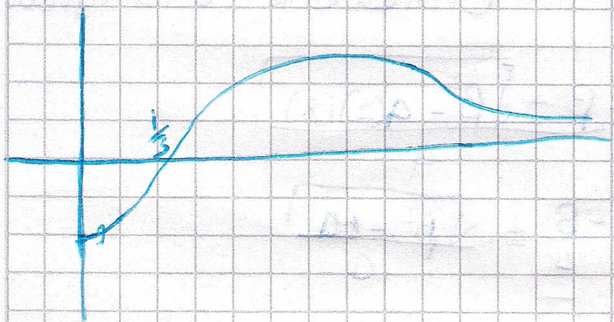
$$0 = -e^{-2t} + 3te^{-2t}$$

$$0 = e^{-2t}(3t-1)$$

$$3t - 1 = 0$$

$$3t = 1$$

$$\boxed{t = \frac{1}{3}}$$



(9) Solucione

$$y'' - 2y' + 2y = \frac{e^x \sec x}{3}$$

Intervalo $(0, \frac{\pi}{8})$

$$m^2 - 2m + 2 \quad 0$$

$$\frac{+2 \pm \sqrt{4-8}}{2}$$

$$m = 1 \pm i$$

$$y_c(x) = e^x \left[c_1 \cos(\cancel{2}x) + c_2 \operatorname{sen}(\cancel{2}x)\right]$$

$$W = \begin{vmatrix} e^x \cos(2x) & e^x \operatorname{sen}(\cancel{2}x) \\ -\cancel{2}\operatorname{sen}(2x)e^x + e^x\cos(\cancel{2}x) & \cancel{2}e^x\cos(\cancel{2}x) + e^x \operatorname{sen}(\cancel{2}x) \end{vmatrix}$$

$$\cancel{2}e^{2x}\cos^2(\cancel{2}x) + \cancel{e^{2x}\cos(2x)\operatorname{sen}(\cancel{2}x)}$$

$$+\cancel{2}e^{2x}\operatorname{sen}^2(\cancel{2}x) - \cancel{e^{2x}\operatorname{sen}(2x)\cos(\cancel{2}x)}$$

$$W = \boxed{\cancel{2}e^{2x}} \qquad w = e^{2x}$$

$$u_1' = \begin{vmatrix} 0 & e^x \operatorname{sen}(x) \\ \dfrac{e^x \sec(x)}{3} & e^x\cos(x) + e^x \operatorname{sen}(x) \end{vmatrix}$$

$$\frac{-\dfrac{\cancel{e^{2x}}\,\dfrac{\operatorname{sen}(x)}{\cos(x)}}{3}}{\cancel{e^{2x}}} \qquad \frac{-\operatorname{sen}(x)}{\cos(x)}$$

$$-\tan(x)$$

$$\int -\frac{\tan(x)}{3}\,dx = -\frac{1}{3}\int \frac{\operatorname{sen}(x)}{\cos(x)}\,dx$$

$u = \cos(x)$

$du = \operatorname{sen}(x)\,dx$

$$\int \frac{1}{u} = \ln(\cos(x))$$

$$u_1 = -\frac{\ln[\cos(x)]}{3}$$

$$u_2' : \begin{vmatrix} e^x\cos(x) & 0 \\ -\operatorname{sen}(x)e^x + e^x\cos(x) & \frac{e^x \sec x}{3} \end{vmatrix}$$

$$= \frac{\frac{e^{2x}}{3}}{e^{2x}} = \int \frac{1}{3}\,dx \qquad u_2 = \boxed{\frac{x}{3}}$$

$$y_p(x) = -\frac{\ln[\cos(x)]}{3} \cdot e^x\cos(x) + \frac{x}{3}e^x \operatorname{sen}(x)$$

$$+ c_1 e^x \cos(x) + c_2 e^x \operatorname{sen}(x)$$

(10)

~~$y(4)$~~ ~~$y_P = ?$~~

$y_P = ?$

$$y^4 + 4y'' = \text{sen } 2t + \cancel{te^t} + \cancel{15}$$

$$m^4 + 4m^2 = 0$$

$$m^2(m^2+4) = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

~~$y_c(x) = C_1 + \ldots$~~

$$y_c(x) = C_1 + xC_2 + C_3\cos(2x) + C_4\,\text{sen}(2x)$$

~~$Ax + Bxe^t + Dx\cos(2x)$~~

$$t\,[A\,\text{sen}(2t) + B\,\text{sen}(2t)] \;\cancel{+ Ex\,\text{sen}(2x)}$$

$$+ (Ct + D)e^t + t^2E$$

(11) $4y'' - Ky' + 9y = 0$

Valor de K para que

$$C.F.S\left\{e^{3t/2},\ xe^{3t/2}\right\}$$

$$m = \frac{K \pm \sqrt{K^2 - 4(9)(4)}}{24} \rightarrow 0$$

$\frac{3}{2}$

$$\boxed{12 = K \pm}$$

(12) $r = -2 \pm 5i$

$((r+2) - 5i)\ ((r+2) + 5i)$

$(r+2)^2 + 25$

$x^2 y'' + 5xy' + 29y = 0$

$r^2 + 4r + 4 + 25$

$r^2 + 4r + 29$

$r^2 - r + 4r + r + 29$

$r(r-1) + 5r + 29$

$a=1 \quad b=5 \quad c=25$

(13) Mayor intervalo PVI unica solucion el

$\ln(x) y'' + \sqrt{10-x}\, y' + \left(\frac{1}{x-5}\right) y = 0$

$y(3) = 1$

$y'(3) = 1$

$\cancel{x > 0}$

$x > 1$

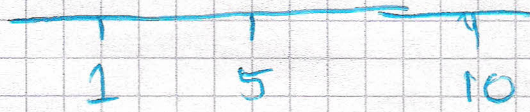
$10 - x \geq 0$

$10 \geq x$

$x \leq 10$

$x - 5 \neq 0$

$x \neq 5$



$\boxed{(1,5)}$

(14)

$\frac{2}{32}$ $\frac{1}{16}$ $1 = 5$

$m = 1/16$

$2 = K1$

$K = 2$

$K = 2$

$\frac{320}{32}$ $\frac{1}{10} = m$ $B = \frac{4}{110}$

$x(0) = -1/2$

$x'(0) = -1$

$10x'' + \frac{4}{10}x' + 2x = 0$

$x'' + \frac{4}{100} + \frac{2}{10} = 0$

PVI $\begin{cases} x'' + \frac{1}{25}x' + \frac{1}{5}x = 0 \\ x(0) = -1/2 \\ x'(0) = -1 \end{cases}$

$x'' + 4x' + 20x = 0$

$m^2 + 4m + 20 = 0$

$\frac{-4 \pm \sqrt{16 - 80}}{2}$

$-2 \pm 4i$

Subamortiguado

$$x(t) = e^{-2t}\left[C_1 \cos(4t) + C_2 \cancel{\operatorname{Sen}}(4t)\right)$$

$$0 = 1\left[C_1 + 0\right)$$

$$\boxed{-\frac{1}{2} = C_1}$$

$$x'(t) = -4e^{-2t}\cancel{\operatorname{Sen}(4t)C_1} - 2e^{-2t}\cos(4t)C_1$$

$$+4e^{-2t}\cos(4t)C_2 - 2e^{-2t}\cancel{\operatorname{Sen}(4t)}$$

$$x'(t) = e^{-2t}\cancel{\cos(4t)} + 4e^{-2t}\cancel{\cos(4t)}C_2$$

$$-1 = 1 + 4C_2$$

$$\cancel{4C_2} = 4C_2 = -2$$

$$\boxed{C_2 = -\frac{1}{2}}$$

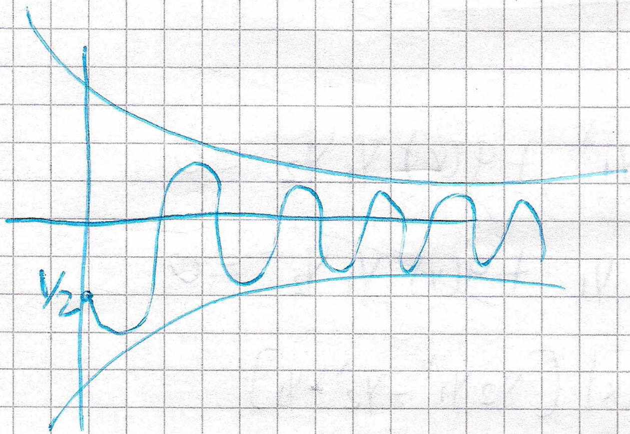
$$\operatorname{Sen}(\omega t + \phi)$$

$$\cos(\omega t)\operatorname{Sen}\phi + \operatorname{Sen}\omega t \cos\phi$$

$\tan = 1$

$$\boxed{\frac{\pi}{4}}$$

$$-\frac{e^{-2x}}{2}\left(\operatorname{Sen} 4t + \frac{\pi}{4}\right)$$



15 $y'' + P(x)y' + q(x)y = 0$

Pruebe que

$$W(y, y_2) = c e^{-\int P(x)dx}$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \quad y_1 y_2' - y_2 y_1' = W$$

$$W' = y_1 y_2'' - y_2 y_1''$$

$$y_1'' + P(x)\,y_1' + q(x)\,y_1 = 0 \cdot y_2$$

$$y_2'' + P(x)\,y_2' + q(x)\,y_2 = 0 \cdot y_2$$

$$y_2 y_1'' + P(x)\,y_2 y_1' + \cancel{q(x)\,y_1 y_2} = 0$$

$$-$$

$$y_2'' y_1 + P(x)\,y_2' y_1 + \cancel{q(x)\,y_1 y_2} = 0$$

$$y_2 y_1'' - y_2'' y_1 + P(x)\left[y_2 y_1' - y_2' y_1\right]$$

$$-w' - P(x)\,w = 0$$

$$w' + P(x)\,w = 0$$

$$\frac{dw}{dx} + P(x)\,w = 0$$

$$\frac{dw}{dx} = -P(x)\,w$$

$$\int \frac{dw}{w} = \int -P(x)\,dx$$

$$\to \ln|w| = \int -P(x)\,dx$$

$$\cancel{e^{c} \cdot \ln|w|} = e^{-\int P(x)\,dx}$$

$$e^{c}\, e^{\ln|w|} = e^{-\int P(x)\,dx}$$

$$w = D\, e^{-\int P(x)\,dx}$$

16

4lb

$$\frac{\cancel{4}^{\,2}}{\cancel{32}_{\,16}} = \frac{1}{8}$$

$s = \frac{1}{2}$

Hooke

$4 = Ks$

$8 = K$

$m = \frac{1}{8}$ $\quad k = 8$

$\beta = \beta$

$x(0) = -1/4$

$x'(0) = 3$

$m^2 + 8\beta m + 64$

$$\frac{-8\beta \pm \sqrt{(8\beta)^2}}{2}$$

$$\frac{x''}{8} + \beta x' + \cancel{8x} = 0$$

$$x'' + 8\beta x' + 64x = 0$$

$x'' + 8\beta x' + 64x = 0$

$x(0) = -1/4$

$x'(0) = 3$

$m^2 + 8\beta m + 64$

$$\frac{-8\beta \pm \sqrt{(8\beta)^2 - 4(64)}}{2}$$

$(8B)^2 - 4(64) = 0$

$64B = 4(64)$

$\boxed{B^2 = 4}$

$B = 2$

$\cancel{m^2 + 82\beta m + 64}$

$\cancel{8}$

$m^2 + 16m + 64$

$$-\frac{16}{2}$$

$m = -8$

$x(t) = C_1 e^{-8t} + C_2 t e^{-8t}$

$$\boxed{\frac{-1}{4} = C_1}$$

$$x'(t) = -8e^{-8t}C_1 + C_2 e^{-8t} \cancel{-8te^{-8t}}$$

$$3 = 2 + C_2$$

$$\boxed{C_2 = 1}$$

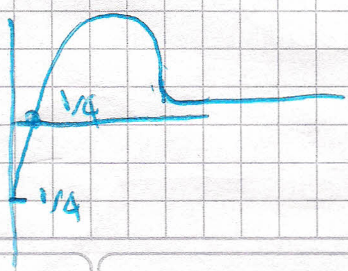
$$x(t) = \frac{-e^{-8t}}{4} + te^{-8t}$$

$$x(t) = e^{-8t}\left(t - \frac{1}{4}\right)$$

$$\boxed{t = \frac{1}{4}}$$ Pasa por cero

$$\boxed{x = \frac{11}{4}e^{-24} \text{ en } t = 3}$$

$$\frac{3}{1} - \frac{1}{4} = \frac{12-1}{4}$$



TERCER CORTE ECUACIONES DIFERENCIALES

1

① Calcule la transformada de laplace de

$$f(t) = t\int_0^t \cosh(\tau)\, d\tau$$

Convolucion

$$\mathcal{L}\{t \; 1 * \cosh(t)\}$$

$$-\frac{d}{ds}\mathcal{L}\{1 * \cosh(t)\}$$

$$-\frac{d}{ds}\left[\frac{1}{s}\cdot\frac{s}{s^2-1}\right]$$

$$-\frac{d}{ds}\left(\frac{1}{s^2-1}\right)$$

$$\boxed{\frac{2s}{(s^2-1)^2}}$$

ⓑ Calcule la transformada inversa de laplace de

$$F(s) = \frac{2(s+5)e^{-3s}}{s^2+10s+29}$$

$$\mathcal{L}^{-1}\left\{\frac{2(s+5)e^{-3s}}{(s+5)^2+4}\right\}$$

$$e^{-5t}\mathcal{L}^{-1}\left\{\frac{2s\; e^{-3s}}{s^2+4}\right\}$$

$$U_3(t)\, e^{-5t}\, 2\cos(2(t-3))$$

$$2U_3(t)\, e^{-5t}\, e^{+15}\cos(2t-6)$$

$$= 2U_3(t)\, e^{-5(t-3)}\cos(2t-6)$$

$$= \boxed{2U_3(t)\, e^{-5t}\cos(2t-6)}$$

② a) Suponga que $F(S) = \mathcal{L}\{f(t)\}$ existe para $S > a \geq 0$. Verifique que si K es una constante positivo, entonces

$$\mathcal{L}^{-1}\{F(KS)\} = \frac{1}{K} f\left(\frac{t}{K}\right)$$

$$\mathcal{L}\,\mathcal{L}^{-1}\{F(KS)\} = \mathcal{L}\left\{\frac{1}{K} f\left(\frac{t}{K}\right)\right.$$

$$F(KS) = \mathcal{L}\left\{\frac{1}{K} f\left(\frac{t}{K}\right)\right\}$$

$$\frac{1}{K}\int_0^{\infty} e^{-St} f\left(\frac{t}{K}\right) dt$$

$$C = \frac{1}{K}$$

$$C\int_0^{\infty} e^{-St} f(Ct)\, dt = \int_0^{\infty} e^{-S\omega/C} f(\omega)\, d\omega = F\left(\frac{S}{C}\right) = F(KS)$$

$\omega = Ct$ $\quad t = \omega/C$

$d\omega = C\,dt$

$\frac{d\omega}{C} = dt$

$$F\left(\frac{S}{C}\right) = \boxed{F(KS)}$$

Ⓑ Resuelva el siguiente PVI

$$y'(t) - \frac{1}{2}\int_0^t (t-\tau)^2\, y(\tau)\, d\tau = t \qquad y(0) = -1$$

$$s\,y(s) - y(0) - \frac{1}{2}\mathcal{L}\{t^2 * y(t)\} = \mathcal{L}\{t\}$$

$$s\,y(s) + 1 - \cancel{\frac{1}{2}}\left(\frac{\cancel{2}}{s^3}\cdot y(s)\right) = -\frac{d}{ds}\left(\frac{1}{s}\right)$$

$$s\,y(s) + 1 - \frac{y(s)}{s^3} = \frac{1}{s^2}$$

$$y(s)\left[s - \frac{1}{s^3}\right] = \frac{1}{s^2} - 1$$

$$y(s)\left[\frac{s^4-1}{s^3}\right] = \frac{1}{s^2} - \frac{1}{1} \qquad \frac{1-s^2}{s^2}$$

$$\cancel{y(s) = \frac{s^3}{s^2(s^4-1)}} \qquad \cancel{\frac{s^3}{s^4-1}}$$

$$y(s) = \frac{(1-s^2)\,s^3}{s^2\,(s^4-1)}$$

$$y(s) = \frac{s\,(1-s^2)}{(s^2-1)(s^2+1)}$$

$$y(s) = \frac{-s\,\cancel{(s^2-1)}}{\cancel{(s^2-1)}(s^2+1)}$$

$$y(s) = -\frac{s}{(s^2+1)}$$

$$\mathcal{L}^{-1}\{y(s)\} = y(t)$$

$$\boxed{y(t) = -\cos(t)}$$

③ $x''(t) + 4x(t) = -\delta(t - \pi/2)$ $\quad x(0) = 0$

$x'(0) = -1$

ⓐ Halle la posicion de la masa en cualquier instante (t).

$$\mathcal{L}\{x(t)\} = X(s)$$

$$s^2 X(s) - s\cancel{x(0)}^{0} - x'(0) + 4X(s) = -e^{-\pi s/2}$$

$$s^2 X(s) + 1 + 4X(s) = -e^{-\pi s/2}$$

$$X(s)[s^2 + 4] = -e^{-\pi s/2} - 1$$

$$X(s) = \frac{-e^{-\pi s/2}}{s^2+4} - \frac{1}{s^2+4}$$

$$\mathcal{L}^{-1}\{X(s)\} = x(t)$$

$$X(t) = -\frac{U_{\pi/2}}{2} \operatorname{sen}(2(t - \pi/2)) - \frac{\operatorname{sen}(2t)}{2}$$

$$X(t) = -\frac{U_{\pi/2}}{2} \operatorname{sen}(2t - \pi) - \frac{\operatorname{sen}(2t)}{2}$$ Antes del golpe

$$X(t) = +\frac{\cancelto{1}{U_{\pi/2}}}{\cancel{2}} \operatorname{sen}(2t) - \frac{\operatorname{sen}(2t)}{2}$$ Despues del golpe

despues de $\pi/2$ la posicion es el equilibrio

(4) Sean $\lambda = -\frac{1}{2} + i$ un valor propio de una matriz

A y $K = \begin{pmatrix} 1 \\ i \end{pmatrix}$ un vector propio de A correspondiente

al valor propio λ

(a) Hallar la solución general (de valores reales del sistema $X'(t) = AX(t)$

$$\begin{pmatrix} 1 \\ i \end{pmatrix} e^{(-\frac{1}{2}+i)t} \; \Big| = \; e^{-t/2} \begin{pmatrix} 1 \\ i \end{pmatrix} e^{it} \; \Big| =$$

$$e^{-t/2} \left[\begin{pmatrix} 1 \\ i \end{pmatrix} \cos(t) + i \operatorname{sen}(t) \right]$$

$$e^{-t/2} \left[\begin{pmatrix} \cos(t) + i \operatorname{sen}(t) \\ i\cos(t) - \operatorname{sen}(t) \end{pmatrix} \right]$$

$$e^{-t/2} \begin{pmatrix} \cos(t) \\ -\operatorname{sen}(t) \end{pmatrix} + i \begin{pmatrix} \operatorname{sen}(t) \\ \cos(t) \end{pmatrix}$$

$$X(t) = e^{-t/2} \left[C_1 \begin{pmatrix} \cos(t) \\ -\operatorname{sen}(t) \end{pmatrix} + C_2 \begin{pmatrix} \operatorname{sen}(t) \\ \cos(t) \end{pmatrix} \right]$$

(b) Hallar la solución general (de valores reales) del sistema $X'(t) = AX + G(t)$ donde

$$G(t) = e^{-t/2} \begin{pmatrix} 0 \\ \csc(t) \end{pmatrix}$$

$$\Psi = \begin{pmatrix} e^{-t/2}\cos(t) & e^{-t/2}\operatorname{sen}(t) \\ -e^{-t/2}\operatorname{sen}(t) & e^{-t/2}\cos(t) \end{pmatrix}$$

$$\det \Psi = e^{-2t/2}\cos^2(t) + 2^{-2t/2}\operatorname{sen}^2(t)$$
$$= e^{-2t/2}$$

$$\Psi^{-1} = e^{t} \begin{pmatrix} e^{-t/2}\cos(t) & -e^{-t/2}\operatorname{sen}(t) \\ e^{-t/2}\operatorname{sen}(t) & e^{-t/2}\cos(t) \end{pmatrix}$$

$$g(t) \cdot \Psi^{-1} = \begin{pmatrix} e^{t/2}\cos(t) & -e^{t/2}\operatorname{sen}(t) \\ e^{t/2}\cos(t) & e^{t/2}\cos(t) \end{pmatrix} \begin{pmatrix} 0 \\ \frac{e^{-t/2}}{\operatorname{sen}(t)} \end{pmatrix}$$

$$\Psi^{-1} g(t) = \begin{pmatrix} 0 & -1 \\ 0 & +\cot(t) \end{pmatrix} \cdot \begin{pmatrix} -1 \\ \cot(t) \end{pmatrix}$$

$$X_p(t) = \Psi \int \begin{pmatrix} -1 \\ \cos(t) \end{pmatrix} dt$$

$$X_p(t) = \Psi \begin{pmatrix} -t \\ \ln|\operatorname{sen}(t)| \end{pmatrix}$$

$$X_p(t) = \begin{pmatrix} e^{-t/2}\cos(t) & e^{-t/2}\operatorname{sen}(t) \\ -e^{-t/2}\operatorname{sen}(t) & e^{-t/2}\cos(t) \end{pmatrix} \begin{pmatrix} -t \\ \ln|\operatorname{sen}(t)| \end{pmatrix}$$

$$\cancel{X_p(t) = \Big(-t\, e^{-t/2}\cos(t)}$$

$$X_p(t) = e^{-t/2} \begin{pmatrix} -t\cos(t) + \operatorname{sen}(t)\ln|\operatorname{sen}(t)| \\ +t\operatorname{sen}(t) + \cos(t)\ln|\operatorname{sen}(t)| \end{pmatrix}$$

$$X(t) = X_c(t) + X_p(t)$$

$$X(t) = e^{-t/2}\Big[c_1 \begin{pmatrix} \cos t \\ -\operatorname{sen} t \end{pmatrix} + c_2 \begin{pmatrix} \operatorname{sen}(t) \\ \cos(t) \end{pmatrix} + t \begin{pmatrix} -\cos(t) \\ \operatorname{sen} t \end{pmatrix}$$

$$\cancel{+\cos}$$

$$+ \ln|\operatorname{sen}(t)| \begin{pmatrix} \operatorname{sen} t \\ \cos t \end{pmatrix}$$

(5) Resuelva la siguiente ecuación integral

(a) $f(t) = 3e^{4t} + 16\int_0^t \tau e^{4\tau} f(t-\tau)\,d\tau$

$\mathcal{L}\{f(t)\} = F(s)$

$F(s) = 3\mathcal{L}\{e^{4t}\} + 16\,\mathcal{L}\{\int_0^t \tau e^{4\tau} f(t-\tau)\,d\tau\}$

$F(s) = \frac{3}{s-4} + 16\,\mathcal{L}\{te^{4t} * f(t)\}$

$F(s) = \frac{3}{s-4} + 16\,\mathcal{L}\{te^{4t}\} \cdot \mathcal{L}\{f(t)\}$

$F(s) = \frac{3}{s-4} + 16(-)\frac{d}{ds}\left[\frac{1}{s-4}\right] \cdot F(s)$

$F(s) = \frac{3}{s-4} + 16\frac{F(s)}{(s-4)^2}$

$F(s) - \frac{16F(s)}{(s-4)^2} = \frac{3}{(s-4)^2}$

$F(s) \cdot \left[1 - \frac{16}{(s-4)^2}\right] = \frac{3}{(s-4)}$

$F(s)\,\frac{(s-4)^2 - 16}{(s-4)^2} = \frac{3}{(s-4)}$

$F(s) = \frac{3(s-4)}{(s-4)^2 - 16}$

$F(s) = \cancel{\frac{3(s-4)}{((s-4)^2-16)(s-4)}}$

$F(t) = 3e^{4t}\cosh(t)$

$F(s) = \frac{3(s-4)}{(s-4)^2 - 16}$

$\mathcal{L}^{-1}\{F(s)\} = F(t)$ → $f(t) = 3e^{4t}\cosh(4t)$

(b) Si f, f' son continuas en o [0, ∞] y son de orden exponencial entonces demuestre que:

$$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0)$$

$$\int_0^{\infty} e^{-st} f'(t)\, dt$$

$u = e^{-st}$ $du = -se^{-st}$

$dv = f'(t)$ $v = f(t)$

$$e^{-st} f(t)\Big|_0^{\infty} + \int_0^{\infty} f(t)\, s e^{-st}\, dt$$

$$\mathcal{L}\{f'(t)\} = -f(0) + s \boxed{\int_0^{\infty} f(t)\, e^{-st}\, dt} \rightarrow \mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f'(t)\} = -f(0) + s\,\mathcal{L}\{f(t)\}$$

(6) En los siguientes literales complete

a) Se sabe que 2 es un valor propio repetido de la matriz $A = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$ y que todos los vectores propios asociados con este valor propio son múltiplos escalares. Entonces la solución general del sistema

$\rightarrow \xi \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 3-2 & 1 \\ -1 & 1-2 \end{pmatrix}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$P_1 + P_2 = -2 \qquad \begin{pmatrix} 0 \\ -2 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ -2 \end{pmatrix}$$

$$P_2 = -P_1 - 2$$

$$X(t) = e^{2t}\left[C_1\begin{pmatrix} -2 \\ 2 \end{pmatrix} + C_2\left[\begin{pmatrix} -2 \\ 2 \end{pmatrix}t + \begin{pmatrix} 0 \\ -2 \end{pmatrix}\right]\right]$$

ⓑ Sea f una funcion continua y de orden exponencial en $[0,\infty)$ si $1 * f = 1-1\cos(t)$ entonces $f(t)$

$$\mathcal{L}\{1 * f\} = \mathcal{L}\{1\} - \mathcal{L}\{\cos(t)\}$$

$$\frac{1}{s}\cdot F(s) = \frac{1}{s} - \frac{s}{s^2+1}$$

$$F(s) = 1 - \frac{s^2}{s^2+1}$$

$$F(s) = \frac{s^2+1-s^2}{s^2+1}$$

$$F(s) = \frac{1}{s^2+1}$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$f(t) = \boxed{\operatorname{Sen}(t)}$$

y $\mathcal{L}\{U_{2\pi} f(t)\} = e^{-2\pi s} \text{sen}(t-2\pi)$

$= e^{-2\pi s} \mathcal{L}\{\text{sen}(t-2\pi)\}$

$\downarrow$

$\text{Sen}(t)\cos(2\pi) - \cancel{\text{Sen}(2\pi)\cos(t)} \to 0$

$= e^{-2\pi s} \mathcal{L}\{\text{sen}(t)\}$

$= \boxed{e^{-2\pi s} \dfrac{1}{s^2+1}}$

c) Si $F(s) = \dfrac{e^{-2\pi s}}{s^2+25}$ entonces su transformada inversa es

$\mathcal{L}^{-1}\{F(s)\} = F(t)$

$F(t) = U_{2\pi}(t) \dfrac{1}{5} \text{Sen}(5(t-2\pi))$

$\boxed{F(t) = U_{2\pi}(t) \dfrac{1}{5} \text{Sen}(5t-10\pi)}$

7

a) Halle la solucion general del sistema

$$X'(t) = \begin{pmatrix} -2 & 4 \\ -2 & 2 \end{pmatrix} X$$

$$\begin{pmatrix} -2-\lambda & 4 \\ -2 & 2-\lambda \end{pmatrix} = 0$$

$$(-2-\lambda)(2-\lambda)+8=0$$

$$-4+\cancel{2\lambda}-\cancel{2\lambda}+\lambda^2+8=0$$

$$\lambda^2+4=0$$

$$\lambda^2 = \pm 2i$$

$$\begin{pmatrix} -2-2i & 4 \\ -2 & 2-2i \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(-2-2i)v_1 + 4v_2 = 0$$

$$v_2 = \frac{-(-2-2i)v_1}{4}$$

$$v_2 = \frac{(2+2i)v_1}{4}$$

$$v_2 = \frac{(1+i)v_1}{2}$$

$$\begin{pmatrix} 1 \\ \frac{1+i}{2} \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1+i \end{pmatrix} e^{2it} \qquad \begin{pmatrix} 2 \\ 1+i \end{pmatrix} \cos(2t) + i\,\text{sen}(2t)$$

$$2\cos(2t) + 2i\,\text{sen}(2t)$$

$$\cos(2t) + i\,\text{sen}(2t) + i\cos(2t) - \text{sen}(2t)$$

$$\begin{pmatrix} 2\cos(2t) \\ \cos(2t)-\text{sen}(2t) \end{pmatrix} + i\begin{pmatrix} 2\,\text{sen}(2t) \\ \text{sen}(2t)+\cos(2t) \end{pmatrix}$$

$$X(t) = C_1\begin{pmatrix} 2\cos(2t) \\ \cos(2t)-\text{sen}(2t) \end{pmatrix} + C_2\begin{pmatrix} 2\,\text{sen}(2t) \\ \text{sen}(2t)+\cos(2t) \end{pmatrix}$$

DD MM AA

(b) Considere el sistema no homogeneo

$$X'(t) = AX(t) + \begin{pmatrix} 3 \\ 1 \end{pmatrix} e^{2t}$$

Donde A es una Matriz 2x2 con entradas constant si se sabe que

$\Psi = \begin{pmatrix} 2e^t & e^{2t} \\ e^t & e^{2t} \end{pmatrix}$ Es una matriz fundamental des sistema homogeneo asociado entonces encuentre la Solucion general de sistema homogeneo dado en $(-\infty, \infty)$

$$X_c(t) = C_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\det \psi = 2e^{3t} - e^{3t} = e^{3t}$$

$$\Psi^{-1} = e^{-3t} \begin{pmatrix} e^{2t} & -e^{2t} \\ -e^{t} & 2e^{t} \end{pmatrix} = \begin{pmatrix} e^{-t} & -e^{-t} \\ -e^{-2t} & 2e^{-2t} \end{pmatrix} = \Psi^{-1}$$

$$g(t) \cdot \psi^{-1} = \begin{pmatrix} e^{-t} & -e^{-t} \\ -e^{-2t} & 2e^{-2t} \end{pmatrix} \begin{pmatrix} 3e^{2t} \\ e^{2t} \end{pmatrix} = \begin{pmatrix} 3e^t - e^t \\ -3 + 2 \end{pmatrix} = \begin{pmatrix} 2e^t \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 2e^t & e^{2t} \\ e^t & e^{2t} \end{pmatrix} \begin{pmatrix} 2e^t \\ -t \end{pmatrix} = \begin{pmatrix} 4e^{2t} - te^{2t} \\ 2e^{2t} - te^{2t} \end{pmatrix} = X_p(t)$$

$$X_p = \psi \int \begin{pmatrix} 2e^t \\ -1 \end{pmatrix} dt$$

$$X_p = \Psi \begin{pmatrix} 2e^t \\ -t \end{pmatrix}$$

$X(t)$?

$$X(t) = c_1 e^{t}\begin{pmatrix}2\\1\end{pmatrix} + c_2 e^{2t}\begin{pmatrix}1\\1\end{pmatrix} + e^{2t}\begin{pmatrix}4\\2\end{pmatrix} - te^{2t}\begin{pmatrix}1\\1\end{pmatrix}$$

$$X(t) = c_1 e^{t}\begin{pmatrix}2\\1\end{pmatrix} + c_2 e^{2t}\begin{pmatrix}1\\1\end{pmatrix} + e^{2t}\left[-\begin{pmatrix}1\\1\end{pmatrix}t + \begin{pmatrix}4\\2\end{pmatrix}\right]$$

8

a) Resuelva el siguiente PVI

$$ty'' - 3ty' - 3y = 0 \qquad y(0) = 0$$
$$y'(0) = 2$$

$$\mathcal{L}\{y(t)\} = y(s)$$

$$\mathcal{L}\{ty''\} - 3\mathcal{L}\{ty'\} - 3\mathcal{L}\{y\} = 0$$

$$(-1)\frac{d}{ds}\left[s^2 y(s) - \cancel{s\,y(0)}^{\to 0} - \cancel{y'(0)}^{\to 2}\right] - 3(-1)\frac{d}{ds}\left[s\,y(s) - \cancel{y(0)}^{\to 0}\right]$$

$$-3y(s)$$

$$(-1)\frac{d}{ds}\left[s^2 y(s) - 2\right] - 3(-1)\frac{d}{ds}\left[s\,y(s)\right] - 3(y(s)) = 0$$

$$(-1)\left[2s\,y(s) + s^2 y'(s)\right] - 3(-1)\left[y(s) + s\,y'(s)\right] - 3y(s) = 0$$

$$-2s\,y(s) - s^2 y'(s) + \cancel{3y(s)} + 3s\,y'(s) - \cancel{3y(s)} = 0$$

$$-s^2 y'(s) + 3s\,y'(s) - 2s\,y(s) = 0$$

$$y'(s)\left[-s^2 + 3s\right] - y(s)(2s) = 0$$

$$y'(s) - y(s)\frac{2s}{[-s^2+3s]} = 0$$

$$y'(s) + y(s)\frac{\cancel{s}2}{\cancel{s}(s-3)} = 0$$

$$y'(s) + y(s)\frac{2}{s-3} = 0$$

$$y'(s) = \frac{-2y(s)}{s-3}$$

$$y' = \frac{-2y}{s-3}$$

$$\frac{dy}{ds} = \frac{-2y}{s-3}$$

$$\int\frac{dy}{y} = -2\int\frac{ds}{s-3}$$

$$\ln(y) = -2\ \ln(s-3) + c$$

$$(e)\ \ln(y) = \ln(s-3)^{-2} + c\ (e)$$

$$y(s) = (s-3)^{-2}\cdot e^{c}$$

$$K$$

$$y(s) = K\frac{1}{(s-3)^2}$$

$$y'(t) = K\left[e^{3t} + \frac{te^{3t}}{3}\right]$$

$$2 = K[1]$$

$$\boxed{K=2}$$

$$Ke^{3t}\ \mathcal{L}\ y\ \frac{1}{s^2}\ y$$

$$Kte^{3t} = y(t)$$

$$y(t) = Kte^{3t}$$

$$\boxed{y(t) = 2te^{3t}}$$

b) Si f es continua por tramos en $[0,\infty)$, de orden exponencial y periodica con periodo T entonces pruebe que

$$F(S) = \mathcal{L}\{f(t)\} = \frac{1}{1-e^{sT}} \int_0^T e^{-st} f(t)\,dt$$

$$\mathcal{L}\{f(t)\} = \int_0^T e^{-st} f(t)\,dt + \int_T^\infty e^{-st} f(t)\,dt$$

$w = t - T$

$dw =$

$$= \int_0^T e^{-st} f(t)\,dt + \int_0^\infty e^{-s(w+T)} f(w+T)$$

$$+ \int_0^\infty e^{-sw} e^{-sT} f(w+T)$$

$$+ e^{-sT} \underbrace{\int_0^\infty e^{-sw} f(w)\,dw}_{F(s)}$$

$$F(S) - e^{-sT}F(S) = \int_0^T e^{-st} f(t)\,dt$$

$$F(S)\left[1 - e^{-sT}\right] = \int_0^T e^{-st} f(t)\,dt$$

$$\boxed{F(S) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt}$$

(9) Rellene

a) $r=i$ es un valor propio de una matriz $A \in \mathbb{R}^{2\times 2}$ si $K = \begin{pmatrix} 1 \\ 1+i \end{pmatrix}$ es un vector propio de A asociada al valor propio r entonces la solución general del sistema $X'(t) = AX(t)$

$$\begin{pmatrix} 1 \\ 1+i \end{pmatrix} e^{it} \quad \frac{1}{1} \begin{pmatrix} 1 \\ i+i \end{pmatrix} \quad \cos(t) + i\,sen(t)$$

$$\cos(t) + i\,sen(t)$$

$$\cos(t) + i\,sen(t) + i\cos(t) - sen(t)$$

$$X(t) = C_1 \begin{pmatrix} \cos(t) \\ \cos(t) - sen(t) \end{pmatrix} + C_2 \begin{pmatrix} sen(t) \\ sen(t) + \cos(t) \end{pmatrix}$$

b) $\mathcal{L}\{3t\, U_{5\pi}(t) \cos(t - 5\pi)\}$

$$(-1)3 \frac{d}{ds}\left[e^{-5\pi s} \frac{s}{s^2+1} \right]$$

$$-3$$

(c) $F(S) = \frac{5e^{-7\pi S}}{5S-2}$

$\mathcal{L}^{-1}\{F(S)\} = F(t)$

$F(t) = \frac{5}{5} U_{7\pi}\ e^{2t/5}$

$= \boxed{U_{7\pi}(t)\ e^{\frac{2(t-7\pi)}{5}}}$

(10)

a) Halle la Solucion general del sistema

$$X'(t) = \begin{pmatrix} -1 & 3 \\ -3 & 5 \end{pmatrix} \bar{X}(t)$$

Puede usar que $r=2$ es un valor propio de Multiplicidad 2 de la matriz asociada al sistema

$$\begin{pmatrix} -1-2 & 3 \\ -3 & 5-2 \end{pmatrix} \qquad \begin{pmatrix} -3 & 3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-3V_1 + 3V_2 = 0$

$-V_1 + V_2 = 0$

$V_2 = V_1 \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} -3 & 3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4/3 \end{pmatrix}$$

$$-3P_1 + 3P_2 = 1$$

$$3P_2 = 1 + 3P_1$$

$$P_2 = \frac{1}{3} + P_1$$

$$X(t) = e^{2t}\left[C_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}t + \begin{pmatrix} 1 \\ 4/3 \end{pmatrix}\right]\right]$$

(b) considere el sistema no homogeneo

$$X'(t) = AX(t) + \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$$

Si se sabe que $\Psi \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix}$

$$\det \Psi = e^{-t} + 4e^{-t}$$

$$\Psi^{-1} = \begin{pmatrix} 5e^{3t} & -5e^{3t} \\ 20e^{-2t} & 5e^{-2t} \end{pmatrix}$$

$$\det \Psi = 5e^{-t}$$

$$\Psi^{-1} = 5e^{t}\begin{pmatrix} e^{2t} & -e^{2t} \\ 4e^{-3t} & e^{-3t} \end{pmatrix}$$

$$\frac{1}{5}\begin{pmatrix} e^{3t} & -e^{3t} \\ 4e^{-2t} & e^{-2t} \end{pmatrix}\begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$$

$$\frac{1}{5}\begin{pmatrix} e^{t} + 2e^{4t} \\ 4e^{-4t} - 2e^{-4t} \end{pmatrix}$$